**1.1.1**. Give an example where a combination of three nonzero vectors in is the zero vector. Then write your example in the form . What are the shapes of , , and ?

**Sol**.

**1.1.2**. Suppose a combination of the columns ofequals a different combination of those columns. Write that as. Find two combinations of the columns ofthat equal the zero vector (in matrix language, find two solutions to).

**Sol**. where

**1.1.3**. (Practice with subscripts) The vectors, , ... , are in-dimensional space, and a combinationis the zero vector. That statement is at the vector level.

(1) Write that statement at the matrix level. Use the matrixwith the's in its columns and use the column vector

(2) Write that statement at the scalar level. Use subscripts and sigma notation to add up numbers. The column vectorhas components , , ... , .

**Sol**. (1)

(2)

**1.1.4**. Supposeis the 3 by 3 matrix ones(3,3) of all ones. Find two independent vectorsandthat solveand. Write that first equation(with numbers) as a combination of the columns of. Why don't I ask for a third independent vector with ?

**Sol**. , only two free variables.

**1.1.5**. The linear combinations ofandfill a plane in.

(a) Find a vectorthat is perpendicular toand. Thenis perpendicular to every vectoron the plane:

.

(b) Find a vectorthat is not on the plane. Check that.

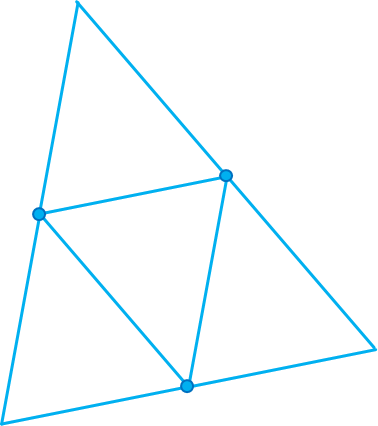
**Sol**. (a)

(b) is not on the plane. .

**1.1.6**. If three corners of a parallelogram are,, and, what are all three of the possible fourth corners? Draw two of them.

**Sol**. The other three possible points are

andrepeated.



**1.1.7**. Describe the column space of. Describe the nullspace of: all vectorsthat solve. Add the "dimensions" of that plane (the column space of) and that line (the nullspace of):

dimension of column space + dimension of nullspace = number of columns

**Sol**. Ifandare linearly independent, , and.

Ifandare linearly dependent and at least one ofandis nonzero, for instance,,

and, and.

If, , and.

**1.1.8**. is a representation of the columns ofin the basis formed by the columns ofwith coefficients in. Ifis 3 by 3, write downandand.

**Sol**.

**1.1.9**. Suppose the column space of anbymatrix is all of. What can you say about? What can you say about? What can you say about the rank?

**Sol**. , ,

**1.1.10**. Find the matricesandcontaining independent columns ofand:

**Sol**. .

**1.1.11**. Factor each of those matrices into. The matrixwill contain the numbers that multiply columns ofto recover columns of. This is one way to look at matrix multiplication:times each column of.

**Sol**.

**1.1.12**. Produce a basis for the column spaces ofand. What are the dimensions of those column spaces – the number of independent vectors? What are the ranks ofand? How many independent rows inand?

**Sol**.

is the number of independent vectors ofwhich has three independent columns.

, .

**1.1.13**. Create a 4 by 4 matrixof rank 2. What shapes areand?

**Sol**.

**1.1.14**. Suppose two matricesandhave the same column space.

(a) Show that their row spaces can be different.

(b) Show that the matrices(basic columns) can be different.

(c) What number will be the same forand?

**Sol**. (a) &

(b)

(c)

**1.1.15**. If, the first row ofis a combination of the rows of. Which part of which matrix holds the coefficients in that combination – the numbers that multiply the rows ofto produce row 1 of?

**Sol**.

**1.1.16**. The rows ofare a basis for the row space of. What does that sentence mean?

**Sol**.

**1.1.17**. For these matrices with square blocks, find. What ranks?

**Sol**.

**1.1.18**. If, what are thefactors of the matrix?

**Sol**.

**1.1.19**. "Elimination" subtracts a numbertimes rowfrom row: a "row operation." Show how those steps can reduce the matrixin Example 4 to(except that this row echelon formhas a row of zeros). The rank won't change!

**Sol**.

**1.1.20**. Show that equation produces in

**Sol**.

**1.1.21**. The rank-2 example in the text produces . Choose rows 1 and 2 directly from to go into . Then from , find the 2 by 2 matrix M that produces . Fractions enter the inverse of matrices: Inverse of a 2 by 2 matrix

**Sol**.

**1.1.22**. Show that formula breaks down if : dependent columns.

**Sol**. If , which does not exist.

**1.1.23**. Create a 3 by 2 matrix with rank 1. Factor into and .

**Sol**.

**1.1.24**. Create a 3 by 2 matrix with rank 2. Factor into and .

**Sol**. where